Discrete convexity and package utilities

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In our paper with V.Danilov and K.Murota in 2001 it was shown that the issue of existence of equilibrium in
economies with indivisible goods and money is related to discrete convexity. Classes of discrete convexity are characterized by unimodularity of the allowable directions of one-dimensional demand sets.

The class of graphical unimodular system can be put in relation with a nicely interpretable economic property of utility functions, the Gross Substitutability property. The question is still open as to what could be the possible, challenging economic interpretations and relevant examples of demand structures that correspond to other classes of discrete convexity. We consider an economy populated with agents having a taste for complementarity; their utilities are generated by compounds of specific items grouped in ""packages.

Simple package-utilities translate in a straightforward fashion the fact that the items forming a package are complements. General package-utilities are obtained as the convolution (or aggregation) of simple package-utilities. We prove that if the collection of packages of items, that generates the utilities of agents in the
economy, is unimodular then there exists a competitive equilibrium. Since any unimodular set of vectors can be
implemented as a collection of 0-1 vectors
(V. Danilov, V. Grishukhin and me, 2009), we get examples of demands for each class of discrete convexity.